Active Roof System

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Engineering Analysis

Document

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Contents

1.0 Average Solar Radiation

Before actual calculations could begin, we needed to research the solar radiation found in the Flagstaff area. The average radiation values for each month are shown in Figure 1 below [1]:

Season	Average Solar Radiation per Month [kWh/m^{2}day]			Average per Season [kWh/m^{2}day]
Fall	*Aug*	*Sep*	*Oct*	**6.43**
	7.05	6.65	5.6	
Winter	*Nov*	*Dec*	*Jan*	**3.93**
	4.25	3.6	3.95	
Spring	*Feb*	*Mar*	*Apr*	**6.32**
	5	6.25	7.7	
Summer	*May*	*Jun*	*Jul*	**8.47**
	8.65	9.25	7.5	

Average Fall & Winter = 5.18 [kWh/m^{2}day]
Average Spring & Summer = 7.39 [kWh/m^{2}day]

Figure 1: Average Solar Radiation [kWh/m^{2}day] for Fall/Winter & Spring/Summer

However, in order to use the data shown above in any of the following calculations or simulations presented in this report, it had to be converted from $\frac{kWh}{m^2 day}$ to $\frac{W}{m^2}$. This was done by using the dimensional analysis shown below:

$$1\,\frac{kWh}{m^2 day} * \frac{1000\ W}{1\ kw} * \frac{1\ day}{8\ hours\ of\ Sun} = 1\,\frac{W}{m^2}$$

The reason why the radiation has been divided by 8 hours in a day rather than 24 hours is because it is more accurate to divide by 8 due to the fact that there is on average approximately 8 hours of sunlight per day. The converted radiation values are shown in Figure 2 on the following page:

Season	Average Solar Radiation per Month [W/m^2]			Average per Season [W/m^2]
Fall	*Aug*	*Sep*	*Oct*	**804.17**
	881.25	831.25	700	
Winter	*Nov*	*Dec*	*Jan*	**491.67**
	531.25	450	493.75	
Spring	*Feb*	*Mar*	*Apr*	**789.58**
	625	781.25	962.5	
Summer	*May*	*Jun*	*Jul*	**1058.33**
	1081.25	1156.25	937.5	

Average Fall & Winter = 647.92 [W/m^2]
Average Spring & Summer = 923.96 [W/m^2]

Figure 2: Average Solar Radiation [W/m^2] for Fall/Winter & Spring/Summer

So during the fall months of August, September and October, the average solar radiation is 804.17 W/m^2. The winter months of November, December and January have an average solar radiation of 491.67 W/m^2. The spring months of February, March and April have an average solar radiation value of 789.58 W/m^2, which is close to the value for the fall months. The summer months of May, June and July have the highest average solar radiation value at 1058.33 W/m^2. To calculate the irradiation value, the average between the fall and winter months as well as the average between the spring and summer months were used. These were 647.9 W/m^2 and 923.9 W/m^2 respectively.

2.0 Average Outside Temperature

A similar process to the solar radiation calculations were used to find the average outside temperature for the Flagstaff area, and these values are shown in Figure 3 below [2].

Season	Average High Temperature per Month [^oF]			Average per Season [^oF]
Fall	*Sep*	*Oct*	*Nov*	**49.67**
	37	62	50	
Winter	*Dec*	*Jan*	*Feb*	**43.67**
	43	43	45	
Spring	*Mar*	*Apr*	*May*	**58.67**
	50	58	68	
Summer	*Jun*	*Jul*	*Aug*	**79.00**
	78	81	78	

Average Fall & Winter = 46.67^{o}F

Average Spring & Summer = 68.83^{o}F

Figure 3: Average Temperature for Fall/Winter and Spring/Summer

The fall and winter values were fairly close together. Ranging from September to February, the average fall temperature is 49.67^{o}F and the average winter temperature is 43.67^{o}F. The spring and summer values were further apart. Ranging from March to August, the average spring temperature is 58.67^{o}F and the average summer temperature is 79.00^{o}F. For our calculations, the average between the fall and winter months as well as the spring and summer months was determined. These were 46.67^{o}F and 68.83^{o}F respectively

3.0 Average Convection Coefficients

In order to complete the Transient Conduction calculations, which will be discussed in the next section of this report, first the average convection coefficients of the air above the roof of the prototypes must be calculated. A convection coefficient is a value which represents how well heat is able to transfer into that specific fluid at a given temperature.

In the case of our prototype the type of convection which we are concerned with is the nature convection in the air about the roof. The roof of the prototypes can be modeled as a horizontal plate with an upper hot surface, as soon in Figure 4 below [3]:

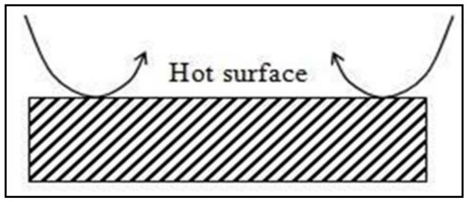

Figure 4: Diagram of Natural Air Flow off of a Horizontal Plate with Upper Hot Surface

The issue that arose while trying to find the average convection coefficient (h_{avg}) for each prototype during each season group was that in a value for the roof surface temperature (T_s) was needed to determine the properties needed to calculate h_{avg}. So then "h" had to be calculated using an iterated process with the following steps:

1st: Guess a roof surface temperature (T_s)

2nd: Calculate h_{avg} using guessed T_s

3rd: Calculate the T_s using h_{avg}

4th: If needed run the program again with a new guessed T_s value

For the 4^{th} step in the process above the newly guessed T_s value was estimated based on how close the calculated value of T_s is to the guessed value.

In order to make this iterative calculation process easier and more time efficient than calculating each step by hand, a Matlab program was written and used. This program used to calculate the h_{avg} and T_s values has been included in this report as Appendix A.

To calculate the T_s values using the h_{avg} values the following emissivity values were used: for $\varepsilon_{\text{black paint}} = 0.92$ [4], $\varepsilon_{\text{white paint}} = 0.99$ [4], and $\varepsilon_{\text{Polished Aluminum}} = 0.92$ [5] (for reflective roof panels). Another set of values which were needed in order to calculate the T_s values were the percent of solar radiation which is estimated to be reflected away from the roof by the reflective roof panels on the passive and active prototype. It should be noted that since the control roof will have no reflected panels, 0% of the solar radiation will be reflected during both season groups. Figure 5 on the next page shows the idealized percent of solar radiation reflection of the active prototype for each seasonal group and the estimated percent of solar radiation reflection of the passive prototype:

Prototype	Fall/Winter	Spring/Summer	
Active	0	100	*Ideal*
Passive	35	65	*Estimated*

Figure 5: Percent of Solar Radiation Reflection of the Active and Passive Prototypes for Each Season Group

3.1 Equations to Calculate the Average Convection Coefficients

The first step in calculating the h_{avg} value is to calculate (the dimensionless) Nusselt Number (Nu_L) for a horizontal plate with an upper hot surface, and in order to do that the following variable corresponding to the approximated average temperature (discussed in the next paragraph) need to be found: the Grashof Number (Gr_L), the Prandtl Number (Pr), the thermal conductivity (k), and the kinematic viscosity (v). All these values but Gr_Lcan be found in Table A.4 in Appendix A of the textbook Fundamentals of Heat Transfer and Mass Transfer [3]. An equation will be needed to calculate Gr_Lwhich will be discussed later.

After guessing a T_s value the next step is to calculate the film temperature ($T_f = \frac{T_s+T_\infty}{2}$ [3]), where T_∞ is the value of the average outside temperatures for each season calculated in section 2. Using this T_f values for the propertiesPr, k, and v can be found by linear interpolation from the table previously discussed.

From there the Gr_Lnumber may be calculated by using the following equation: $Gr_L = \frac{1}{v^2}[gB(T_s - T_\infty)L^3]$ [3] where $g = 9.81\frac{m}{s^2}$ (gravity), $B = \frac{1}{T_f}$ [3] (since we are dealing with air), and $L = 4.5$ft (which is the smallest length of the roof based on the interior dimensions of the Prototype).

The Rayleigh Number ($Ra_L = Gr_L * Pr$ [3]) can now be calculated and then it will be needed to find the Nu_L value. For the configuration of a horizontal plate with an upper hot surface, $Nu_L = 0.54{Ra_L}^{1/4}$[3]. It is now that the h_{avg} value may be calculated by setting $Nu_L = \frac{\text{havg}*\text{L}}{k}$ [3] and then solving for h_{avg}.

It now in the iterative process that the initially guessed T_s value has to be checked by calculating the T_s that would result by using the h_{avg} value the guessed T_s was used to calculate. To do this, a simple energy rate balance can be applied for the roof: $\Delta\dot{E}_{in} - \Delta\dot{E}_{out} = 0$. From there, for the case of any of the roof systems the energy rate balance expands to the following equation:

$\alpha G - h_{avg}(T_s - T_\infty) - \varepsilon\sigma\left(T_s{}^4 - T_\infty{}^4\right) = 0$ [3]. In this equation α = the percent of solar radiation reflected, G = the amount of solar radiation (presented in section 1), ε = emissivity of the roof surface, and σ = the Stephan-Boltzmann constant = $5.67 * 10^{-8} \frac{W}{m^2K}$ [3].

The energy rate balance above can then be solved for the positive T_s value, and if this T_s value is not within a few tenths of the guessed value of T_s used to calculate h_{avg} then the calculation process must be ran again with a guessed T_s value closer to the T_s value which was calculated in the previous iteration. Examples of the calculated outcome values of this iteration are shown in the table of the next section.

3.2 Calculated Average Convection Coefficients

The calculations from the iteration of h_{avg} calculations for the ***control*** prototype during the fall/winter and spring/summer season groups are shown in Figure 6 below, and the row of values which is bolded is the value of h_{avg} which was selected to be used in future calculations.

1. Winter/Fall Control

Ts Guess [^oF]	h [w/m^2K]	Ts Calc [^oF]
80	4.795307	148.1162
120	6.094375	139.0496
135	6.417086	137.012
137	**6.456344**	**136.769**

2. Spring/Summer Control

Ts Guess [^oF]	h [w/m^2K]	Ts Calc [^oF]
120	5.323518	193.4888
190	6.793277	182.867
183	6.686961	183.5834
184	**6.702513**	**183.479**

Figure 6: T_s Guesses, Resulting h Values and Calculated T_s Values for Control Prototype

The calculations from the iteration of h_{avg} calculations for the ***passive*** prototype during the fall/winter and spring/summer season groups are shown in Figure 7 below:

1. Winter/Fall Passive

Ts Guess [^oF]	h [w/m^2K]	Ts Calc [^oF]
120	6.094375	111.4592
111	5.868005	112.6292
112	**5.895079**	**112.487**

2. Spring/Summer Passive

Ts Guess [^oF]	h [w/m^2K]	Ts Calc [^oF]
150	6.088725	158.9738
158	6.251314	156.7886
157	**6.231706**	**157.0478**

Figure 7: T_s Guesses, Resulting h Values and Calculated T_s Values for Passive Prototype

The calculations from the iteration of h_{avg} calculations for the ***active*** prototype during the fall/winter and spring/summer season groups are shown in Figure 8 on the next page:

1. Winter/Fall Active

Ts Guess [^oF]	h [w/m^2K]	Ts Calc [^oF]
120	6.094375	142.1366
130	6.315323	140.6372
138	6.475679	139.5752
139	**6.494823**	**139.4492**

2. Spring/Summer Active

Ts Guess [^oF]	h [w/m^2K]	Ts Calc [^oF]
140	5.864779	68.8298
80	3.289482	68.8298
69	**0.8202**	**68.8298**

Figure 8: T_s Guesses, Resulting h Values and Calculated T_s Values for Active Prototype

A table summarizing all the h_{avg} values, that have been selected to be used in the next section's calculations and the T_s values which lead to calculating the h_{avg} value, is included below (Table 1):

Table 1: Summary of T_s Guesses, Resulting h Values and Calculated T_s Values for All Prototypes

Prototype	Seasons	Ts Guess [^oF]	Ts Calc [^oF]	h [w/m^2K]
Control	Winter/Fall	137.00	136.77	6.46
	Spring/Summer	184.00	183.48	6.70
Passive	Winter/Fall	112.00	112.49	5.90
	Spring/Summer	157.00	157.05	6.23
Active	Winter/Fall	139.00	139.45	6.49
	Spring/Summer	69.00	68.83	0.82

It is worth pointing out here that the T_s of the active prototype during the spring/summer season group is the same temperature which we calculated to be the average outside/ambient temperature during this season group. This makes sense because if, ideally, 100% of the solar radiation during the spring/summer is reflected away from the reflected panels on the roof of the active roof, then the T_s should be equal to the outside temperature.

4.0 Transient Conduction

In transient conduction, a solid object is changing temperature as time a function of time, so in order to use this type of heat transfer model we had to make a very important assumption. The assumption are that due to the small ceiling height of the inside of our prototypes (0.65ft) that there will be no internal circulation (advection) and that means that it can be assumed that the main mode of heat transfer through the air within the prototype will be by conduction rather than convection. This is an important assumption because the interior and roof (cork insulation) of the prototype need to be modeled as one solid object in order to apply this type of heat transfer analysis, and since we assumed that heat will only be transferred by conduction we are able to model the interior and the cork insulation as one solid object.

How we model the roof as one solid object as one solid object is by evaluating the properties of each of the material types (air and cork) at room temperature which approximately equals 300K, and then taking a weighted average of these values based on the thickness of each material. The following property values for air and cork were taken from Table A.3 and Table A.4 [3], respectively: $\rho_{cork} = 120\frac{kg}{m^3}$, $k_{cork} = 0.039\frac{W}{m\cdot K}$, $Cp_{cork} = 1800\frac{kJ}{kg\cdot K}$, $\rho_{air} = 1.1614\frac{kg}{m^3}$, $k_{air} = 0.0263\frac{W}{m\cdot K}$, and $Cp_{air} = 1.007\frac{kJ}{kg\cdot K}$. Where in this context ρ is the density and Cp is the specific heat of the material at that given temperature. So for example, the weighted average for k was found by using the following equation: $= \frac{t_{cork}k_{cork}+t_{air}k_{air}}{t_{cork}+t_{air}}$, where $t_{cork} = 3\left(\frac{3}{32}in\right)\left(\frac{1\,ft}{12\,in}\right) = 0.0234ft$ (because 3 layers of $\frac{3}{32}$in cork used for ceiling) and $t_{air} = height\ of\ interior = 0.65ft$. The same type of calculation was used to find the weighted average of ρ and Cp. Table 2 below shows the values of these weighted average properties:

Table 2: Calculated Values of Needed Properties using a Weighted Average

Property	Symbol	Average	Units
Density	ρ	37.05	kg/m^3
Thermal Conductivity	k	0.03	W/m·K
Specific Heat	Cp	1246.5	J/kg·K

Now the Biot Number (Bi) can be calculate using the formula $Bi = \frac{h_{avg}L_c}{k}$ [3] where h_{avg} is the values shown in the previous section and L_c is the characteristic length of the roof which is $L_c = \frac{V_{olume}}{A_{surface}} = \frac{(4.5*4.5*0.65)ft}{(4.5*4.5)ft} = 0.65ft$. From here the Bi number is used to pull two particular constant values off of Table 5.1 [3]: ζ_1 and C_1.

For this case the approximate solution is found for the mid-plane of the modeled solid object and the equation used was $\frac{T_o-T_\infty}{T_i-T_\infty} = C_1\cos\left(-\zeta_1{}^2F_o\right)$, where T_∞ is the outside/ambient air presented in section 2, $T_i = 70^oF$ which is the initial temperature of the inside of the prototype, $T_o = 75^oF$ which is the temperature which our team has decided is the temperature which interior of buildings become uncomfortable, and $F_o$is the Fourier Number and is equal to $\frac{\alpha_{Fo}*t}{L_c{}^2}$

where $\alpha_{Fo} = \frac{k}{\rho * Cp}$ (using the weighted values calculated) and t is the time (in seconds) it takes the modeled solid to reach the T_o temperature if starting at the T_i temperature.

Needless to say this calculation, if done by hand, is rather labor intensive so our team created a Matlab program that would calculate the time variable based on the input values of h_{avg}, ζ_1 and C_1 input by the user. This program is included in this report as Appendix B.

These calculated time values for each prototype during each season group is shown in Table 3 below:

Table 3: Time for the Lumped Solid of the Prototype to Reach 75°F from 70°F

	Time to Reach 75°F from 70°F (min)	
Prototype	**Winter/Fall**	**Spring/Summer**
Control	2.657	80.392
Passive	2.660	80.672
Active	2.656	105.747

The values in the table above prove that this calculation process was done correctly and valid because the time it takes the active roof system to reach $T_o = 75^oF$ in the spring/summer season group is larger than the other two prototypes, and that is accurate because the active roof system has reflective panels which would, ideally, block all the solar radiation from reaching the roof surface. The same kind of thing can be seen with the winter/fall season group for the active roof system, but this time the interior of the prototype will reach $T_o = 75^oF$ the fastest because the panels are now reflecting 0% of the solar radiation and therefore all of it is being absorbed by the black roof below.

The important thing to note about the time values calculated in the Table 3 above is that they were based on our assumption that heat will be transferred by conduction rather than convection, and air has a much lower rate of heat transfer by conduction than it does by convection. So as will be proved in the following section, there is air circulation with the inside of the prototypes, so the heat will be transferred through the air by convection. However, these time values were necessary to calculate because they give us an idea of how fast the inside of the box will heat up during winter/fall compared to spring/summer, and since the calculated time values for winter/fall are all around 2.5minutes our team has concluded that a heating system is not necessary as part of the air conditioning component of these prototypes. This decision was based off of the fact that since the heat is being transferred by convection rather than conduction than the interior of the box is going to heat up to $T_o = 75^oF$ much faster than in 2.5 minutes as was calculated under the assumption that the heat transfer was by conduction, so even during the winter/fall months, the prototypes will on, average, during testing only need an air conditioning unit to cool down the interior.

5.0 Checking for Internal Circulation

In order to check for circulation within the interior of the prototypes, first the model must be identified as an enclosure with natural convection occurring from within. From there the $\bar{T} = \frac{T_1+T_2}{2}$ value can be calculated using $T_1 = T_{hot} = T_{celing}$ which we chose to be varying temperature from 70^oF to 90^oF because these are the highest ceiling temperatures we expect to have inside the prototypes during testing and $T_2 = T_{cold} = T_{floor}$ which we chose to be either 70^oF or 75^oF because 70^oF is our ideal internal temperature and 75^oF is the temperature at which our air conditioning unit will turn on.

From here the following values are evaluated for air at the $\bar{T}$ temperature where the properties are listed in Table A.3 [3]: Pr and v. From there the following is calculated: $Gr_L = \frac{1}{v^2}[gB(T_s - T_\infty)L^3]$ [3] where just like before, $= 9.81\frac{m}{s^2}$, $B = \frac{1}{T_f}$ [3], and $L = 4.5$ft. From there the Rayleigh Number can be calculated just like before: $Ra_L = Gr_L * Pr$ [3].

In order to efficiently complete this calculation of multiple T_1 and T_2 values a Matlab program was created and has been included in this report as Appendix C. The calculated Ra_L values for various T_1 and T_2 values are shown in Table 4 below:

Table 4: Ra_L Values for Varying Ceiling Temperatures

	Ra_L Number ($*10^9$)for Different $T_{ceiling}$ (^oF)				
T_{floor} (^oF)	70	75	80	85	90
70	0	0.702	1.375	2.020	2.638
75	-	0	0.673	1.319	1.935

For an enclosure, if the Ra_L value is less than 1708 than the buoyancy forces of the air are unable to overcome the resistance of the viscosity of the fluid in the enclosure (in this case air), and therefore there is no natural circulation (natural convection) of the fluid within the enclosure. However, as can be seen in Table 4 above, all of the calculated Ra_L values for every expected T_1 and T_2 values is well above 1708 so therefore there will always be some kind of natural circulation of the air within the prototypes if the ceiling and floor are not at the same temperature (which in testing is highly unlikely).

6.0 Estimating the Temperature of the A/C Air

So in order to keep the interior of our prototype at the desired, constant temperature of 70^oF, our team needs to know at approximately what temperature would the air blown in from the air conditioning unit have to be in order for the interior of the prototype to be cooled from 75^oF (our chosen uncomfortable temperature) to 70^oF.

To begin, the density of air (ρ_{air}) at room temperature is $1.1614\frac{kg}{m^3}$ (as shown in section 4 above). Therefore the mass of the air that would normally be contained in the interior of our prototype is found by $m_{air} = wlh\rho_{air} = (4.5 * 4.5 * 0.65)ft^3\left(\frac{0.0283168\,m^3}{ft^3}\right)\left(1.1614\frac{kg}{m^3}\right) = 0.4329kg$. For the sake of calculation we are assuming that half of the hot air naturally goes out the vents build into the prototype walls, so the $m_{1hot} = m_{1cold} = m_1 = 0.21644kg$, where m_{1hot} is the mass of the hot air (air already inside the prototype) and m_{1cold} is the mass of the air conditioned air been blown into the prototype. Also, $m_2 = 2m_1 = m_{air} = 0.4329kg$, which is the total mass of the resulting ideal gas mixtures.

So to start the analysis a basis energy balance of a closed system is performed: $\Delta U + \Delta KE + \Delta PE = Q_{in} - W_{out}$, which then leads to $U_1 - U_2 = m_1(u_{1hot} + u_{1cold}) - m_2u_2$. This equation can be then solved for u_{1cold}: $u_{1cold} = \frac{m_2u_2 - m_1u_{1hot}}{m_1} = 2(u_2) - u_{1hot}$.

Where u_{1hot} and u_2 can be found by linearly interpolating the values of the internal energy of air at the corresponding $T_1 = 75^oF = 297.039K$ and $T_2 = 70^oF = 294.261K$ from Table A- 22 in the textbook Fundamentals of Engineering Thermodynamics [6]. The needed internal energy values for interpolation are as follows:

- At $T = 290K$: $u = 206.91\frac{kJ}{kg}$
- At $T = 300K$: $u = 214.07\frac{kJ}{kg}$
- At $T = 295K$: $u = 210.49\frac{kJ}{kg}$

So then $u_{1hot} = \frac{(297.039-295)K}{(300-295)K}(214.07 - 210.49)\frac{kJ}{kg} + 210.49\frac{kJ}{kg} = 211.9499\frac{kJ}{kg}$, and then $u_2 = \frac{(294.261-290)K}{(295-290)K}(210.49 - 206.91)\frac{kJ}{kg} + 206.91\frac{kJ}{kg} = 209.9609\frac{kJ}{kg}$.

Entering these values into the u_{1cold} equation created leads to the following:

$$u_{1cold} = 2\left(209.9609\frac{kJ}{kg}\right) - 211.9499\frac{kJ}{kg} = 207.9719\frac{kJ}{kg}$$

However, we want the temperature value of T_{1cold} so we can do this by finding the corresponding temperature to this u_{1cold} value by using Table A-22 [6] again, and as a matter of fact the internal energy and temperatures which are needed for this linear interpolation have actually already been cited above. So to get T_{1cold} , the following formula is used:

$$T_{1cold} = \frac{(207.919-206.91)\frac{kJ}{kg}}{(210.49-206.91)\frac{kJ}{kg}}(295-290)K + 290K = 291.4831K = 64.999^oF$$

Therefore, in order to cool down the air within the inside of the prototype from 75^oF to 70^oF with half of the hotter air naturally escaping through the vents built into the prototype, the cold air blown from the air conditioning unit would have to be approximately equal to or les than 65^oF.

7.0 Computer Simulated Fluid Modeling

For the computer simulation we used the 4.5ft x 4.5ft x 0.65ft dimensions that we calculated previously for the interior dimensions to represent the air inside the model. The average airspeed of industrial duct systems is 10m/s [7]. The calculated value for inlet temperature is about 65^oF, we chose to use 62^oF in the simulations because we simulated the worst-case scenario and did not adjust values for the use of panels, which will lower the radiation, thus reducing the interior temperature of the building. With the two simulations, one for the fall/winter and one for spring/summer, each run with 50 iterations. We found that with the cooling system that we plan on using we will be able to keep the model at a temperature of about 77^oF in the summer and 71^oF in the winter. These temperatures simulation can be seen in Figures 9 and 10. Using the panels will result in being able to reach temperatures closer to the target temperature of 70^oF.

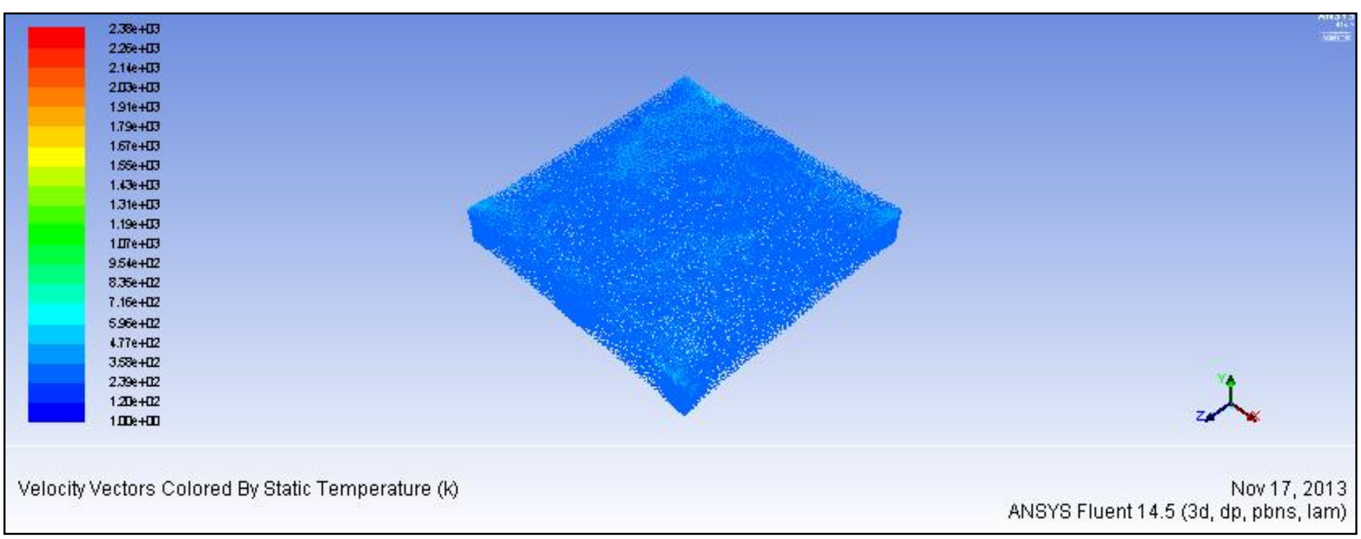

Figure 9: Average Interior Summer Temperature Simulation

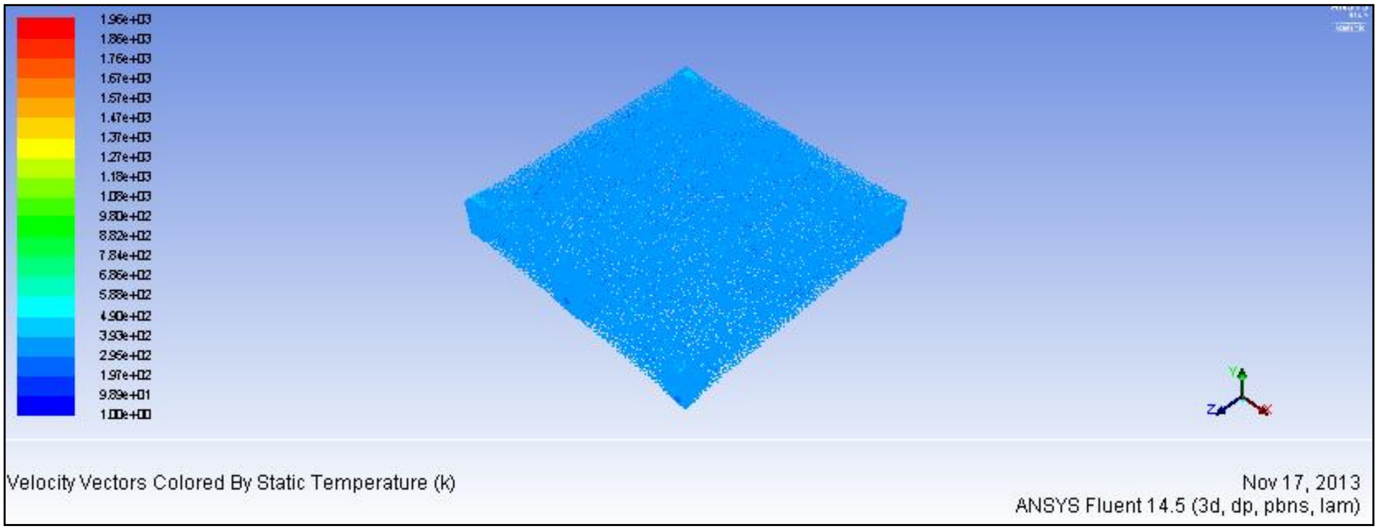

Figure 10: Average Interior Winter Temperature Simulation

After researching various fans for the inlets we found that placing small individual fans will supply the necessary amount of air so we will not be required to use larger fans [8]. We have found fans that would supply ample airflow which will be supplied to each inlet to cool the building to the desired temperature. These mini fans have an airflow of 10 cfm which is approximately 10 m/s which is the desire airflow.

We plan to install fan systems for the each of the six inlet hose components to produce the required air flow into the interior model system. The fan system is responsible for maintaining interior temperatures at $70^{o}F$. The simulation process of the Ansys program showed that the fans are required to supply air mass into the interior system at a rate of 10 m/s at 62 degrees Fahrenheit at each of the six one inch diameter inlet hose components. We also plan to have four outlet hose systems which will not consist any type of fanning system but rather will produce a natural air flow rate equivalent to the air mass flow intake of the interior system.

8.0 Conclusion

In order to keep the interior temperature of our prototypes at a comfortable 70^{o}F, six inlets, each with individual fans, will be installed on one side of the prototypes. There will be four outlets to circulate the moving air throughout the prototype. With the active or passive system in place, the average interior temperature should be around 77^{o}F in the summer and 71^{o}F in the winter. Based on the heat transfer calculations, the air conditioning temperature should be about 62^{o}F and will be blown in at 10 m/s. This extra cooling will drop the temperature down to our goal of 70^{o}F. Based on these calculations, a heating system will not be required for the prototypes during the winter months. With these conditions and fan implementations, the interior of our prototypes should be able to maintain a 70^{o}F interior temperature.

9.0 References

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Appendix A: Matlab Code to Calculate the Average Convection Coefficient (h_{avg})

```
% Date Created: 11/15/2013
% Presentation Date: 11/18/2013
% Authors: Capstone Team 06
% Head Editor: Krysten Whearley
% Program Description: Calculating the Convection Coefficient (h) for the
%                      Free Convection above the Prototype Roof

function main
% List Givens
To = (5/9)*(75-32)+273;          % To=75F (Need all temps in K)
Ti = (5/9)*(70-32)+273;          % Ti=70F
L = 4.5*0.3048;                  % Length of roof ft to m
rho = 5.67*10^(-8);              % W/m2K4 (Stephan-Boltzman Constant)
e_black = 0.92;                  % Emissivity Black Roof (Passive & Active)
e_reflect = 0.05;                  % Emissivity Polished Aluminum (Refelective Panels)
e_white = 0.99;                  % Emissivity White Roof (Control)

% List Assumptions
seasons=input('Enter 1(Fall/Winter) & 2(Summer/Spring): ');
     % Input initital guess for Ts of roof to calc h for air above roof
Ts_guess_F = input('Guess of Ts: ');
Ts_guess = (5/9)*(Ts_guess_F-32)+273; % Convert T(F) to T(K)
  if seasons==1
      T_inf = (5/9)*(46.67-32)+273;    % for Fall/Winter: T_inf=46.67F
      G_irrad = 647.9167;              % W/m2
  elseif seasons==2
      T_inf = (5/9)*(68.83-32)+273;    % for Summer/Spring: T_inf=68.83F
      G_irrad = 923.9583;              % W/m2
  end

% Calculate Properites of the air using Tf using Table A-4 in Fundamentals
% Heat and Mass Transfer Edition 7
Tf = (Ts_guess+T_inf)/2;
fprintf('Tf = %f K\n',Tf);
   B = 1/Tf;
   if Tf<=300
       T_low=250;
       T_high=300;
       v_low = 11.44*10^(-6);
       v_high = 15.89*10^(-6);
       k_low = 22.3*10^(-3);
       k_high = 26.3*10^(-3);
       Pr_low = 0.720;
       Pr_high = 0.707;
   elseif Tf<=350
       T_low=300;
       T_high=350;
       v_low = 15.89*10^(-6);
       v_high = 20.92*10^(-6);
       k_low = 26.3*10^(-3);
       k_high = 30*10^(-3);
       Pr_low = 0.707;
       Pr_high = 0.700;
   end
   v = ((Tf-T_low)/(T_high-T_low))*(v_high-v_low)+v_low;
   k = ((Tf-T_low)/(T_high-T_low))*(k_high-k_low)+k_low;
   Pr = ((Tf-T_low)/(T_high-T_low))*(Pr_high-Pr_low)+Pr_low;

%-----------------------------------------------------------------------%
% Calculating h
Gr_L = abs((1/(v^(2)))*(9.81*B*(Ts_guess-T_inf)*L^(3)));
```

```
Ra_L = Gr_L*Pr;
  % For Free Convection of Horizontal Plate with Upper Hot Surface
  if Ra_L<10^(7)
      Nu_L = 0.54*Ra_L^(1/4);     % Eqn 9.30
  elseif Ra_L>10^(7)
      Nu_L = 0.15*Ra_L^(1/3);     % Eqn 9.31
  end
h = Nu_L*(k/L);

%-------------------------------------------------------------------------%
% Calculating Ts values
roof = input('Enter 1(Control), 2(Passive) & 3(Active): ');

if roof ==1     % For Control Roof
   alpha_cont = 1;     % for all seasons percent absorbtivity
   % aTs+bTs^(4)=c Put into Wolfram to solve for Ts
     a = h;
     b = e_white*rho;
     c = (alpha_cont*G_irrad)+(h*T_inf)+(e_white*rho*T_inf^(4));
   fprintf('%f*x+(%e)*x^(4)=%f\n',a,b,c);    % Input into Wolfram Alpha
   Ts_K = input('Input Ts(K): ');
   fprintf('h = %f W/m2K\n',h);
   Ts_F = (9/5)*(Ts_K-273)+32;                 % Convert Ts(K) to Ts(F)
   fprintf('Ts = %f deg F\n',Ts_F);

elseif roof==2               % For Passive Roof
   if seasons==1
       alpha_pass = 0.65;     % for Fall/Winter percent absorbtivity
       e_pass = e_black;
   elseif seasons==2
       alpha_pass = 0.35;     % for Spring/Summer percent absorbtivity
       e_pass = e_reflect;
   end
   % aTs+bTs^(4)=c Put into Wolfram to solve for Ts
     a = h;
     b = e_pass*rho;
     c = (alpha_pass*G_irrad)+(h*T_inf)+(e_pass*rho*T_inf^(4));
   fprintf('%f*x+(%e)*x^(4)=%f\n',a,b,c);    % Input into Wolfram Alpha
   Ts_K = input('Input Ts(K): ');
   fprintf('h = %f W/m2K\n',h);
   Ts_F = (9/5)*(Ts_K-273)+32;                % Convert Ts(K) to Ts(F)
   fprintf('Ts = %f deg F\n',Ts_F);

elseif roof==3            % For Active Roof
   if seasons==1
       alpha_act = 1;      % for Fall/Winter percent absorbtivity
       e_act = e_black;
   elseif seasons==2
       alpha_act = 0;      % for Spring/Summer percent absorbtivity
       e_act = e_reflect;
   end
   % aTs+bTs^(4)=c Put into Wolfram to solve for Ts
     a = h;
     b = e_act*rho;
     c = (alpha_act*G_irrad)+(h*T_inf)+(e_act*rho*T_inf^(4));
   fprintf('%f*x+(%e)*x^(4)=%f\n',a,b,c);    % Input into Wolfram Alpha
   Ts_K = input('Input Ts(K): ');
   fprintf('h = %f W/m2K\n',h);
   Ts_F = (9/5)*(Ts_K-273)+32;                % Convert Ts(K) to Ts(F)
   fprintf('Ts = %f deg F\n',Ts_F);
end

end
```

Appendix B: Matlab Code to Calculate the Time it would Take the Inside of the Prototypes to Reach $T_{uncomfortable}$=75°F using Transient Conduction

```
% Date Created: 11/15/2013
% Presentation Date: 11/18/2013
% Authors: Capstone Team 06
% Head Editor: Krysten Whearley
% Program Description: Calculating the Time it takes the Interior of the
%                      Prototypes to Reach 75 F from 70 F


function main
% List Givens
To = (5/9)*(75-32)+273;          % To=75F (Need all temps in K)
Ti = (5/9)*(70-32)+273;          % Ti=70F

% List Assumptions
seasons=input('Enter 1(Fall/Winter) & 2(Summer/Spring): ');
     % Input initial guess for Ts of roof to calc h for air above roof
  if seasons==1
      T_inf = (5/9)*(46.67-32)+273;  % for Fall/Winter: T_inf=46.67F
  elseif seasons==2
      T_inf = (5/9)*(68.83-32)+273;  % for Summer/Spring: T_inf=68.83F
  end

% Input h value Found from "Capstone_h.m"
h = input('Input h: ');


%Finding Weighted Average Properties of Prototypes
t_c = (3/32)*3*(1/12);   % ft (Thickness of Cork Ceiling Insulation)
t_a = 0.65;              % ft (Height of Inside Ceiling)
  % Values from Table A-3 and A-4 for Cork and Air at T=300K
  dens_c = 120;       % kg/m3
  dens_a = 1.1614;
  k_c = 0.039;        % W/mK
  k_a = 0.0263;
  Cp_c = 1800;        % J/kgK
  Cp_a = 1.007*1000; % kJ/kgK to J/kgK

dens_avg = ((t_c*dens_c)+(t_a*dens_a))/(t_c+t_a);
k_avg = ((t_c*k_c)+(t_a*k_a))/(t_c+t_a);
Cp_avg = ((t_c*Cp_c)+(t_a*Cp_a))/(t_c+t_a);

%-----------------------------------------------------------------------%
%Finding Properties (from Table 5.1 for Plane Wall)
Volume = (4.5*4.5*0.65)*0.028317;    % ft3 to m3
Surf_Area = (4.5*4.5)*0.09290304;    % ft2 to m2
Lc = Volume/Surf_Area;               % Characteristic Length
Bi = (h*Lc)/k_avg;
fprintf('Bi = %f \n',Bi);

 Bi_low = input('Input Bi low: ');
 Bi_high = input('Input Bi high: ');
   L1_low = input('Input L1 low: ');
   L1_high = input('Input L1 high: ');
      L1 = ((Bi-Bi_low)/(Bi_high-Bi_low))*(L1_high-L1_low)+L1_low;
   C1_low = input('Input C1 low: ');
   C1_high = input('Input C1 high: ');
      C1 = ((Bi-Bi_low)/(Bi_high-Bi_low))*(C1_high-C1_low)+C1_low;

%-----------------------------------------------------------------------%
```

```
%Finding Time it Takes for the Interior of Prototype to Reach To=75F
alpha_cond = k_avg/(dens_avg*Cp_avg);      % m2/s
    % Solve Fo from Eqn 5.44
    Fo = (1/(L1^(2)))*log((1/C1)*((To-T_inf)/(Ti-T_inf)));  % in Matlab log=ln
    Fo_abs = abs(Fo);
    % Solve t from Eqn 5.12
    time_s=(Lc^(2)*Fo_abs)/(alpha_cond);
    time_min = time_s/60;
    fprintf('time = %f min\n',time_min);
end
```

Appendix C: Matlab Code to Calculate the Ra_L Numbers for Varying $T_{ceiling}$

```
% Date Created: 11/16/2013
% Presentation Date: 11/18/2013
% Authors: Capstone Team 06
% Head Editor: Krysten Whearley
% Program Description: Calculating the Ra number for the internal convection
%                      of varying ceiling and floor temperatures


% List Givens
T2(1) = (5/9)*(70-32)+273;  % Floor, T1=70F
i= 70;                      % Begin at 70F (Need all temps in K)
count=1;
while i<95
    T1_70(count) = (5/9)*(i-32)+273;  % (Need all temps in K)
    count=count+1;
    i=i+5;
end

T2(2) = (5/9)*(75-32)+273;   % Floor, T1=75F
j=75;                        % Begin at 75F now
count=1;
while j<95
    T1_75(count) = (5/9)*(j-32)+273;
    count=count+1;
    j=j+5;
end

L = 4.5*0.3048;              % Length of roof ft to m

% Calculate Properites of the air using Tf using Table A-4 in Fundamentals
% Heat and Mass Transfer Edition 7
for l=(1:length(T1_70))
T_avg = (T1_70(l)+T2(1))/2;
   B = 1/T_avg;
   if T_avg<=300
       T_low=250;
       T_high=300;
       v_low = 11.44*10^(-6);
       v_high = 15.89*10^(-6);
       Pr_low = 0.720;
       Pr_high = 0.707;
   elseif T_avg<=350
       T_low=300;
       T_high=350;
       v_low = 15.89*10^(-6);
       v_high = 20.92*10^(-6);
       Pr_low = 0.707;
       Pr_high = 0.700;
   end
   v = ((T_avg-T_low)/(T_high-T_low))*(v_high-v_low)+v_low;
   Pr = ((T_avg-T_low)/(T_high-T_low))*(Pr_high-Pr_low)+Pr_low;

Gr_L = abs((1/(v^(2)))*(9.81*B*(T1_70(l)-T2(1))*L^(3)));
RaL_70(l) = Gr_L*Pr;
end


for m=(1:length(T1_75))
T_avg = (T1_75(m)+T2(2))/2;
   B = 1/T_avg;
   if T_avg<=300
       T_low=250;
       T_high=300;
```

```
        v_low = 11.44*10^(-6);
        v_high = 15.89*10^(-6);
        Pr_low = 0.720;
        Pr_high = 0.707;
    elseif T_avg<=350
        T_low=300;
        T_high=350;
        v_low = 15.89*10^(-6);
        v_high = 20.92*10^(-6);
        Pr_low = 0.707;
        Pr_high = 0.700;
    end
    v = ((T_avg-T_low)/(T_high-T_low))*(v_high-v_low)+v_low;
    Pr = ((T_avg-T_low)/(T_high-T_low))*(Pr_high-Pr_low)+Pr_low;

Gr_L = abs((1/(v^(2)))*(9.81*B*(T1_75(m)-T2(2))*L^(3)));
RaL_75(m) = Gr_L*Pr;
end
```